transmission in a perpendicular direction across a crystalline plate cut otherwise than perpendicular to a principal plane, I have since worked out the differential equation (between two parameters) of the lines of curvature of the wave-surface, the discussion of which shows that there are no umbilici out of the principal planes. Hence the four directions determined by equations (15) are the only ones perpendicular to which if a plate be cut one of the images is free from astigmatism.—*October* 1877.

XI. "Notes on Physical Geology.—No. III. On the probable Age of the Continent of Asia and Europe; and on the Absolute Measure of Geological Time." By Rev. S. HAUGHTON, M.D., D.C.L., F.R.S., Professor of Geology in the University of Dublin. Received June 11, 1877.

This paper was withdrawn by the Author after the reading, for correction of a numerical error, and will appear as amended next Session.

The Society then adjourned over the Long Vacation, to Thursday, November 15.

"On the Increase in Resistance to the Passage of an Electric Current produced on certain Wires by Stretching." By HERBERT TOMLINSON, B.A., Demonstrator of Natural Philosophy, King's College, London. Communicated by Prof. W. G. ADAMS, F.R.S. Received November 14, 1876. Read December 21 *.

The object of this iuquiry was (1) to determine the relation between increased resistance to the passage of an electric current and stretching-force; (2) to ascertain how much of the increased resistance in each case is produced by mere increase of length and diminution of section.

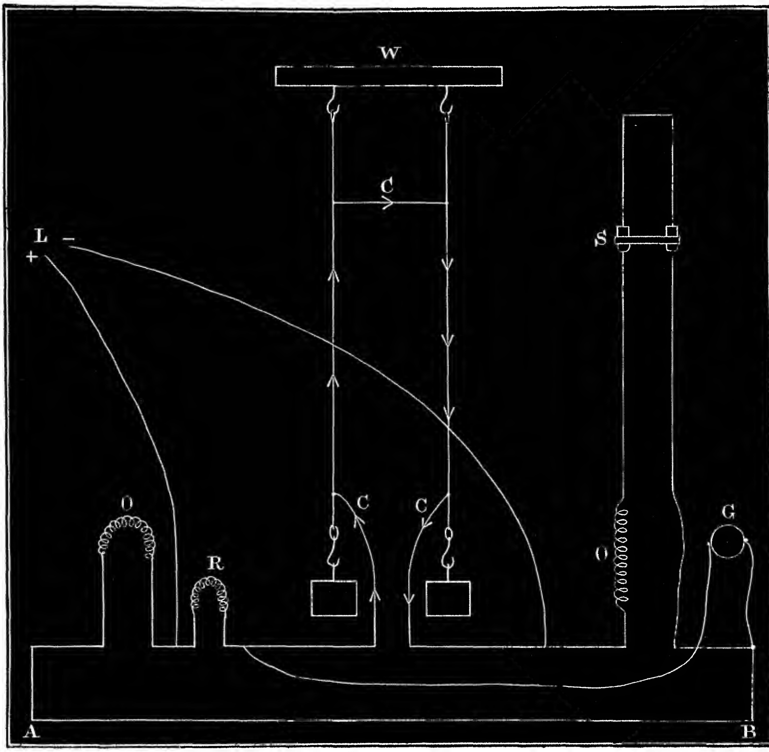
In order to determine the increase of resistance from stretching, the wires were each divided into two parts about 14 feet or more in length. One end of each part was fastened to a stout hook, firmly fixed into a block of wood, the two hooks used being about 8 inches apart, and the block of wood in which they were fixed securely fastened across two strong uprights, which were placed resting against the wall, so that the weights attached to the wires might swing clear of the table on which the uprights were placed. A loop was made at the other end of each part

* See Proc. Roy. Soc. vol. xxv. p. 451.

of the wire, and to this the weights were attached by means of strong hooks. The two parts of the wire were joined at the top, about 2 inches below each hook, by a piece of copper wire, which was securely soldered on to each part of the wire, so as to connect them; and toward the lower extremities of the two parts, about 5 inches above the points of attachment of the weights, two copper wires were soldered so as to connect the wires with a Wheatstone's bridge. The increase of resistance of the wires was measured by means of a sliding scale of platinum wire, divided into millimetre divisions, each equal to ·00166 ohm. As the object was to obtain the temporary and not the permanent increase of resistance (which permanent increase was found more or less with all the wires), weights slightly heavier than those intended to be used were first put on and taken off. Afterwards the wire was balanced as nearly as possible by German-silver wire, without the sliding scale, and then very exactly with the sliding scale, which was connected with one of two resistance-coils of 100 ohms each, which formed the other two sides of the bridge. The weights used were then carefully put on to the wires, and the increase of resistance measured by means of the sliding scale; the weights were next taken off again, and the sliding scale used for balancing once more. If there was any slight difference, as sometimes occurred, between the readings of the sliding scale before the weights were put on and after they were taken off, the mean of the two readings was taken.

In order to secure still greater accuracy, as many as eight or ten trials were frequently made with each particular weight, and the mean of all the trials taken. This precaution was necessary, as there were continual small variations from slight changes of temperature in the air of the room. These variations of temperature caused at first great trouble in the case of iron and steel wires, as the slightest difference of temperature was at once shown by the shifting of the light on the scale of the Thomson's reflecting galvanometer employed. This instrument was so delicate that the warmth of the hand placed 6 inches from the wire caused a perceptible shifting of the light. It was, indeed, partly owing to this difficulty that the observation of the permanent increase from stretching was not attempted. I have, however, since devised a plan for getting rid of this alteration from changes of temperature, and I hope to be able to make other experiments with wires in which the permanent as well as the temporary increase of resistance will be observed. Most of the observations of the steel and iron wires were made at first in the evening, with the doors and windows shut, as it was found almost impossible to take very exact observations in the daytime.

The disposition of the wires is shown in the accompanying sketch. In order to avoid heating the wires by the current, a single Leclanché cell was employed, and an increase of resistance $=\frac{1}{50000}$ of the whole could easily be measured.



W. Wooden block.
L. Leclanché cell.
O, O. Resistances of 100 ohms each.
R. German silver used to balance wire.
C, C, C. Connecting-wires of copper.
S. Sliding scale.
G. Galvanometer.

In this manner 4 pianoforte steel wires, 1 commercial steel wire, 3 iron wires, and 4 brass wires were examined with several different weights.

In the case of all the wires of different sections and materials, it was found that the increase of resistance was exactly proportional to the stretching-weight. In order to show this the increased resistance per lb. weight for the different weights employed is given in the following Table, for the first three steel wires examined, as with them a greater number of different weights were employed than with the other wires :—

	Number of pounds employed to stretch the wire.	Increase per pound, reckoned in divisions of the sliding scale.
Steel (No. 1). Section in square inches determined from loss of weight of a given length of the wire in water at 4° C. = 2231×10^{-7}. Total length of wire employed = 27·46 feet.	2	36·8
	3	36·8
	4	36·9
	5·75	36·8
	8·42	36·8
	9·75	37·0
	12	36·7
	12·5	36·4
		Mean . . 36·78
Steel (No. 2). Section in square inches, 6528×10^{-7}. Total length of wire employed = 27·5 feet.	2·75	12·72
	8	12·52
	16	12·52
	20	12·52
	24	12·52
	28	12·32
		Mean . . 12·52
Steel (No. 3). Section in square inches, 12563×10^{-7}. Total length of wire employed = 27·65 feet.	8	6·07
	12	6·08
	14	6·07
	20	6·05
	28	6·06
	40	6·08
		Mean . . 6·069

Though a smaller number of different weights were employed with the other wires, the proportion of the increase of resistance to the stretching-weight was quite as exact as in the case of the three examples given, never less than three different weights being used, and each of these tried several times.

The increase of resistance which a cubic centimetre of each wire would experience when stretched by a weight of 1 gramme was then calculated, and also the resistance of a cubic centimetre of each wire in ohms determined. The former values will be denoted by h, and the latter by the letter s.

The following Table gives the values of h, s, the section of each wire in square inches, and its specific gravity :—

	Sp. gr. (density of water at 18° C. taken as 1).	Section in square inches, determined from loss of weight in water at 4° C.	Increase of resistance of 1 cub. centim. for a stretching-force of 1 gramme, $=h$.	Resistance of 1 cub. centim. of the wire, $=s$.
Steel.			Ohms.	Ohms.
No. 1	7·8630	2231×10^{-7}	3009×10^{-17}	$1574{\cdot}8\times10^{-8}$
" 2	7·8240	6528×10^{-7}	3159×10^{-17}	$1653{\cdot}1\times10^{-8}$
" 3	7·7945	12563×10^{-7}	3445×10^{-17}	$1882{\cdot}4\times10^{-8}$
" 4	7·8280	21009×10^{-7}	2982×10^{-17}	$1628{\cdot}7\times10^{-8}$
" 5 (Commercial steel)....	7·7072	8477×10^{-7}	3511×10^{-17}	$1847{\cdot}0\times10^{-8}$
Iron.				
No. 1	7·7496	5003×10^{-7}	2557×10^{-17}	$1217{\cdot}6\times10^{-8}$
" 2	7·5300	10239×10^{-7}	2637×10^{-17}	$1200{\cdot}8\times10^{-8}$
" 3	7·6409	19010×10^{-7}	2712×10^{-17}	$1291{\cdot}0\times10^{-8}$
Brass.				
No. 1	8·4879	5781×10^{-7}	1843×10^{-17}	$656{\cdot}7\times10^{-8}$
" 2	8·4984	8782×10^{-7}	1729×10^{-17}	$782{\cdot}2\times10^{-8}$
" 3	8·4965	11327×10^{-7}	1565×10^{-17}	$744{\cdot}8\times10^{-8}$
" 4	8·5048	18258×10^{-7}	1809×10^{-17}	$742{\cdot}9\times10^{-8}$

The values of h, divided by those of s, will give the increase of resistance for a stretching-weight of 1 gramme per unit of resistance. The following Table gives the values of $\frac{h}{s}$ for the different wires:—

Values of $\frac{h}{s}$, or increase of resistance per unit of resistance for a stretching-weight of 1 gramme on a cubic centim. of the material.

Steel.

(1)	$1910{\cdot}4\times10^{-12}$	Mean $=1875{\cdot}5\times10^{-12}$. Greatest departure from mean value less than 2·7 per cent.
(2)	$1910{\cdot}4\times10^{-12}$	
(3)	$1830{\cdot}5\times10^{-12}$	
(4)	$1825{\cdot}5\times10^{-12}$	
(5)	$1900{\cdot}7\times10^{-12}$	

Iron.

(1)	$2100{\cdot}0\times10^{-12}$	Mean $=2132{\cdot}2\times10^{-12}$. Greatest departure from mean about 3 per cent.
(2)	$2196{\cdot}6\times10^{-12}$	
(3)	$2100{\cdot}1\times10^{-12}$	

Brass.

(1)	$2229{\cdot}2 \times 10^{-12}$	Mean $= 2244{\cdot}9 \times 10^{-12}$. Greatest departure from the mean about 8·5 per cent.
(2)	$2211{\cdot}8 \times 10^{-12}$	
(3)	$2101{\cdot}5 \times 10^{-12}$	
(4)	$2435{\cdot}0 \times 10^{-12}$	

It will be seen from the last Table that the values of $\frac{h}{s}$ are as constant as could be expected in the case of wires of the same material, but differ for wires of different materials, being greatest for brass.

The value of Young's modulus for some of the wires was ascertained in the usual manner with the cathetometer. The wires were firmly secured at the top of a staircase, and a mark placed about 3 feet above the point of attachment of the weights at the bottom, so that the stretching of about 21 feet of the wire could be observed and the increase of length for different weights observed. A mark was also made near the top of the wire, in order to examine if there was any yielding of the support to which the upper end of the wire was attached, and for some of the wires a reading was taken both at the top and bottom marks; but as after several trials there was no apparent yielding, in the last experiments made the mark only at the bottom was observed. Weights heavier than those intended to be used were first put on and then taken off again several times; finally a weight was left on the wire sufficient to keep it perfectly straight. Then weights were carefully added, the increase of length observed with the cathetometer; and the weights having been taken off, another reading taken, and if there was any slight difference in the readings before adding the weight and after taking it off, the mean of the two readings was taken. The results agreed very fairly with each other—how much so may be gathered from the following example (two trials with each weight were taken):—

Iron (No. 2).

For a stretching-weight of	A lengthening was observed of
10220 grammes	·0005774 m.m. per gramme.
7710 "	·0005837 " "
17930 "	·0005828 " "
Mean	·0005813 " "

The result may be taken as a fair average of the values obtained with the other wires.

The value of Young's modulus was also ascertained by means of longitudinal vibrations of sound. The two ends of the wire to be examined were securely soldered into two strong flat iron bars, one of the bars was fastened in a vice, firmly secured, whilst the other bar was held in the hand, and the wire pulled with sufficient force to enable a

clear note to be got out of the wire by rubbing it longitudinally with a resined glove. The wire was then clipped in the centre and rubbed again to test the security of the fastenings, as it was found that if the bar in the vice was not well secured the note produced by clipping in the centre was not the octave of the note given out by the unclipped wire, but if the bar were well fastened a clear and perfect octave could be produced.

The number of longitudinal vibrations was ascertained by means of the siren, generally three or four trials of 3 minutes each being taken. In this work I was assisted by Mr. Furse (the assistant to the Professor of Natural Philosophy at King's College), to whom I am much indebted for help in my experiments. The records of the siren were very good indeed, in every case there never being a greater difference than $\frac{2}{10}$ per cent., and in many cases less. But on experimenting with different lengths of the wires the values of the modulus would vary as much as 1 or 2 per cent., so that, though some of the values obtained are recorded, I have chosen the more reliable ones of the cathetometer for calculating the increase of length of each wire when stretched.

In the following Table the amount of lengthening which a cubic centimetre of each wire experiences for a stretching-force of 1 gramme is given; these values will be denoted by $\frac{1}{e}$:—

	Values of $\frac{1}{e}$ obtained from observations with the cathetometer.	Values of $\frac{1}{e}$ obtained from observations on longitudinal vibrations.
Steel.		
(1)	—*	4866×10^{-13}
(2)	5279×10^{-13}	4928×10^{-13}
(3)	5082×10^{-13}	5073×10^{-13}
(4)	—*	
(5)	5665×10^{-13}	
Iron.		
(1)	4896×10^{-13}	
(2)	5938×10^{-13}	
(3)	5435×10^{-13}	
Brass.		
(1)	10120×10^{-13}	

In the case of those wires marked * there was not sufficient length to take good observations with the cathetometer.

Again dividing the values of $\frac{h}{s}$ by those of $\frac{1}{e}$ we shall obtain the increase of resistance per unit of resistance per unit increase of length.

Values of $\frac{h}{s} \div \frac{1}{e}$.

Steel.

(2)	3·619	Mean 3·525
(3)	3·602	
(5)	3·355	

Iron.

(1)	4·289	Mean 3·951
(2)	3·699	
(3)	3·864	

Brass.

(1)	2·203

It would seem that the increase per cent. of resistance for a given lengthening of a wire is greater in iron than in steel, and much greater in both iron and steel than in brass, but that the increase per cent. of resistance per unit of stretching-force employed is greater in brass than in iron, and greater in iron than in steel.

The torsional rigidity of the wires was then ascertained, the vibrators used being similar to those used by Sir William Thomson in his experiments on the rigidity and viscosity of metals (Proc. of Roy. Soc., May 1865), namely, thin cylinders of sheet brass, turned true outside and inside, supported by a thin, flat, rectangular bar. The wire to be tested passed perpendicularly through a hole in the middle of the bar, and was there firmly soldered. The cylinder was tied to the horizontal bar by light threads, so as to hang with its axis vertical; the other end of the wire was securely soldered into a stout iron bar, firmly held in a vice attached to a rigid support. Two experiments with different lengths of wire were made with each wire, the result of each of the two experiments agreeing very closely, as, for instance, we may take as an average example Steel No. 3, in which the torsional rigidities in grammes per square centimetre in the two trials were $783{\cdot}5 \times 10^6$ and $782{\cdot}2 \times 10^6$.

If we assume the wires to be isotropic, we can, from the values of e and the rigidity, which latter value will be denoted by r, obtain the ratio of lateral lineal contraction to longitudinal dilatation; denoting this ratio by σ, we shall obtain (Thomson and Tait's Nat. Phil. p. 521) $\sigma = \frac{e}{2r} - 1$, and therefore easily deduce the increase of resistance that would follow in the case of each wire from mere increase of length and diminution of section, without considering any other alteration of resistance that would result from stretching. Calling ds the increase of resistance that would result from mere increase of length and diminution of section, we can prove that

$$\frac{ds}{s} = \frac{1}{r} \cdot \frac{1}{e}.$$

Again subtracting from the values of $\frac{h}{s}$ those of $\frac{ds}{s}$ we shall obtain the residual alteration of resistance produced by the stretching-force.

[Since making the above experiments I have found that Sir William Thomson has investigated the effects of strain on iron and copper, and states that he attempted to eliminate the effects of elongation and narrowing, and had very nearly established, for iron wire at least, that the augmented resistance due to tension, either temporary or permanent, is a very little more than can be accounted for by the change of form. (Phil. Trans., Feb. 28, 1856, § 152.)]

In the following Table are shown the results obtained:—

	Torsional rigidity in grms. per sq. cent. $=r$.	Ratio of lateral contraction to longitudinal dilatation, $=\sigma$.	Increase of resistance per unit of resistance, resulting from increase of length and diminution of section, $=\frac{ds}{s}$.	Residual increase of resistance per unit of resistance resulting from the stretching.	Mean.
Steel.					
(2)	$746{\cdot}5\times10^{-6}$	·269	$811{\cdot}6\times10^{-12}$	$1098{\cdot}8\times10^{-12}$	$1079{\cdot}6\times10^{-12}$
(3)	$782{\cdot}3\times10^{-6}$	·259	$770{\cdot}1\times10^{-12}$	$1060{\cdot}4\times10^{-12}$	
Iron.					
(1)	771·1	·325	$807{\cdot}6\times10^{-12}$	$1292{\cdot}7\times10^{-12}$	$1257{\cdot}6\times10^{-12}$
(2)	637·2	·321	$975{\cdot}2\times10^{-12}$	$1221{\cdot}4\times10^{-12}$	
Brass.					
(1)	332·5	·486	$995{\cdot}5\times10^{-12}$	$233{\cdot}7\times10^{-12}$	$233{\cdot}7\times10^{-12}$

It would appear from this last Table that the increase of resistance produced by a given stretching-force is, in the case of steel, iron, and brass, not to be accounted for by mere increase of length and diminution of section of the wire; and the residual increase of resistance, which results from subtracting from the whole observed increase that due to mere increase of length and diminution of section, is greater in iron than in steel, and much greater in steel than in brass. In all probability this residual increase is due to the increased distance between the particles of the wire along the line of flow of the current; and perhaps, if we examined the effect of strain in a direction perpendicular to the direction of the current, there would be a diminution of resistance; but I have experiments now in hand by which I hope to show the effect of strain in a direction at right angles to that of the current. In conclusion, I may mention that, in testing for the

increase of resistance from stretching, when the weights were taken off the wire did not immediately attain the resistance which it ultimately settled at, but gradually recovered itself in about three minutes.

This effect was very perceptible with the thicker wires, where heavy weights were employed. I suppose the effect may, at any rate, be partly attributed to the heat which would be generated when the wire recovered its former volume; but I am not at all satisfied that the whole of the effect observed was due to this cause.

The conclusions to be drawn from the experiments are:—

1. That the temporary increase of resistance of a wire when stretched in the same direction as the current is exactly proportional to the stretching-force.

2. That the increase per cent. of resistance when a cube of each material is stretched by the same weight is greater in iron wire than in steel wire, and greater in brass wire than in iron wire; also that this increase is nearly the same for different specimens of the same material.

3. That the increase per cent., when the material is stretched to the same extent, is much greater in iron and steel wires than in brass wire, and is probably greater in iron wire than in steel wire.

4. That there is a residual increase in each case, over and above that which would follow from mere increase of length and diminution of section, that this residual increase is much greater in iron and steel than in brass, and greater in iron than in steel.

November 15, 1877.

Sir JOSEPH HOOKER, C.B., President, in the Chair.

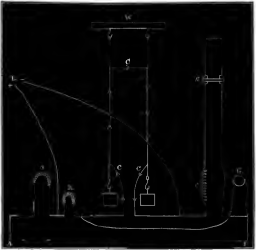
In pursuance of the Statutes, notice of the ensuing Anniversary Meeting was given from the Chair.

Mr. Henry Nottidge Moseley was admitted into the Society.

Mr. Abel, Prof. Carey Foster, Mr. Huggins, Prof. Jevons, and Prof. Parker, having been nominated by the President, were elected by ballot Auditors of the Treasurer's Accounts on the part of the Society.

The Presents received were laid on the table, and thanks ordered for them.

A portrait in gilt frame of Sir John Herschel, by the Danish artist Jensen, presented by Mr. John Evans, F.R.S., and a portrait in gilt



W. Wooden block.
L. Leclanché cell.
O, O. Resistances of 100 ohms each.
R. German silver used to balance wire.
O, O, C. Connecting-wires of copper.
S. Sliding scale.
G. Galvanometer.